

STRING REPRESENTATION OF GAUGE THEORIES

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In this talk, various approaches to the problem of evaluation of the field strength correlators in the $SU(3)$ -gluodynamics, which play the major rôle in the Stochastic Vacuum Model, are reviewed. This is done in the framework of the effective Abelian-projected theories under the various assumptions implied on the properties of the ensemble of Abelian-projected monopoles. In particular, within the assumption on the condensation of the monopole Cooper pairs, the main method of investigation is the string representation of field strength correlators. The calculation of the bilocal field strength correlator in the 3D effective theory, where Abelian-projected monopoles are assumed to form a gas, based on the string representation for the Wilson loop in this theory, is also presented.

1 Introduction. The problem of string representation of gauge theories

The problem of string representation of gauge theories is unambiguously related to the problem of confinement in these theories. Its essence is the quest of a string theory, which is mostly adequate for the description of strings between color objects, which appear in the confining phase of QCD. (Other, Abelian-type, gauge theories possessing the confining phase then serve for probing various approaches to the construction of the string representation of QCD.) Quantitatively, the QCD string can be seen by virtue of the Wilson's picture of confinement¹. It states that the criterion of confinement in QCD is the area law behavior of the Wilson loop

$$\langle W(C) \rangle \equiv \frac{1}{N_c} \left\langle \text{tr} \mathcal{P} \exp \left(ig \oint_C A_\mu^a T^a dx_\mu \right) \right\rangle \xrightarrow{C \rightarrow \infty} e^{-\sigma |\Sigma_{\min}(C)|}. \quad (1)$$

Here, σ is the so-called string tension, *i.e.* the energy density of the QCD string. The latter one is nothing else, but a tube formed by the lines of chromoelectric flux, which appears between two color objects propagating along the contour C . When these objects try to move apart from each other,

the QCD string stretches and prevents that, thus ensuring their confinement. According to Eq. (1), during its propagation, such a string sweeps out the surface of the minimal area for a given contour C , $\Sigma_{\min}(C)$. Due to the dimensional reasons,

$$\sigma \propto \Lambda_{\text{QCD}}^2 = \frac{1}{a^2} \exp \left[-\frac{16\pi^2}{\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)g^2(a^{-2})} \right]$$

with $a \rightarrow 0$ standing for the distance UV cutoff (*e.g.* the lattice spacing). Clearly, all the coefficients in the expansion of σ in powers of g^2 vanish, which means that the QCD string is an essentially nonperturbative object.

Owing to this observation, it is nowadays commonly argued that the area law is well saturated by the strong background fields in QCD. Around those, there additionally exist perturbative fluctuations of the QCD vacuum ², which excite the string. This means that these fluctuations enable the string to sweep out with a nonvanishing probability not only $\Sigma_{\min}(C)$, but also an arbitrary surface $\Sigma(C)$, bounded by C . Therefore, the final aim in constructing the string representation of QCD is a derivation of the formula $\langle W(C) \rangle = \sum_{\Sigma(C)} e^{-\mathcal{S}[\Sigma(C)]}$. Here, $\sum_{\Sigma(C)}$ and $\mathcal{S}[\Sigma(C)]$ stand for a certain sum

over string world sheets and a string effective action, both of which are yet unknown in QCD. Clearly, this formula is just a 2D analogue of the well known representation for the propagator of a point-like particle, which is subject to external forces and/or propagates in external fields. In particular, the rôle of the classical trajectory of such a particle is played within this analogy by $\Sigma_{\min}(C)$. However, it unfortunately turns out to be difficult to proceed from the 1D case, where the sum over paths is universal (*i.e.* depends only on the dimension of the space-time), and the world-line action is known for a wide class of potentials and external fields, to the 2D case under study. In the next Section, we shall discuss various field-theoretical models, where the string effective action and/or the measure in the sum over world sheets are either already postulated *a priori* or can be derived.

2 Field-theoretical models and their string representations

As a natural origin for the QCD string effective action serves the Stochastic Vacuum Model (SVM) of the QCD vacuum ³ (for a review see *e.g.* ²). Within this model, the effective action reads

$$\mathcal{S}[\Sigma(C)] = 2 \int_{\Sigma(C)} d\sigma_{\mu\nu}(x) \int_{\Sigma(C)} d\sigma_{\mu\nu}(x') D\left(\frac{(x-x')^2}{T_g^2}\right). \quad (2)$$

Here, D is one of the two coefficient functions, which parametrize the bilocal gauge-invariant correlator of the field strengths, and T_g is the so-called correlation length of the QCD vacuum, *i.e.* the distance at which the non-perturbative part of the D -function decreases as $e^{-|x|/T_g}$. According to the existing lattice data, $T_g \simeq 0.13$ fm for the $SU(2)$ -case ⁴, and $T_g \simeq 0.22$ fm for the $SU(3)$ -case ⁵ (see also Ref. ⁶ for related investigations and Ref. ⁷ for reviews). Clearly, Eq. (2) means that the function D plays the rôle of the propagator of a nonperturbative gluon, which propagates between the points x and x' lying on the string world sheet $\Sigma(C)$.

The simplest Abelian-type theory possessing the property of confinement is the 3D compact QED ⁸. In this theory, however, this phenomenon is caused by stochastic magnetic fluxes penetrating through the contour C , which are generated by magnetic monopoles. Consequently, the string effective action in this case can be derived, rather than postulated. The dual (disorder) scalar field, describing the grand canonical ensemble of monopoles in this theory, acquires a nonvanishing (magnetic) mass due to the Debye screening in the Coulomb gas of monopoles. Being proportional to the square root of the Boltzmann factor of a single monopole, this mass is essentially nonperturbative, *i.e.* depends nonanalytically on the electric coupling constant. It can be shown ⁹ that in the approximation of a dilute monopole gas, the string effective action has the form similar to Eq. (2) with the function D replaced by the propagator of the massive dual boson.

Another Abelian-type theories where confinement takes place are the so-called Abelian-projected theories ¹⁰. There, within the so-called Abelian dominance hypothesis ¹¹, one gets as an effective IR theory, corresponding to the $SU(N_c)$ -gluodynamics, the $[U(1)]^{N_c-1}$ magnetically gauge-invariant dual theory with monopoles. Further investigations of such a theory depend on the particular form of the average over the monopole ensemble. One of the possibilities is to treat this ensemble as a Coulomb gas, which is a good approximation in 3D. In particular, in the $SU(2)$ -case one then arrives just at the compact QED, discussed above. Another possibility, more appropriate in 4D, is to demand the condensation of the monopole Cooper pairs, which leads to the dual Abelian Higgs type theory. In such a theory, confinement can be described as the dual Meissner effect ¹², *i.e.* it is due to the formation of the dual Nielsen-Olesen strings ¹³. By introducing external particles, electrically

charged *w.r.t.* the Cartan subgroup, one can see that the string effective action in this theory again has the form (2) with the function D replaced by the propagator of a dual vector boson. Contrary to the Debye mechanism realized in the monopole gas, the dual bosons acquire now their magnetic mass due to the Higgs mechanism.

It is further possible to perform an expansion of the nonlocal action (2) in powers of the derivatives *w.r.t.* the world-sheet coordinates. Clearly, in the SVM-case such an expansion is equivalent to the T_g -one, whereas in the case of Abelian-projected theories this is just an expansion in the inverse powers of the magnetic mass of the dual boson. Then, as the two leading terms of this gradient expansion one gets the Nambu-Goto term and the so-called rigidity term ¹⁴, whose coupling constants in the SVM-case read ¹⁵ $\sigma = 4T_g^2 \int d^2 z D(z^2)$ and $\frac{1}{\alpha_0} = -\frac{T_g^4}{4} \int d^2 z z^2 D(z^2)$, respectively. The positive string tension σ as well as the negative sign of the other coupling constant ensure the stability of string configurations in the models under consideration (see Ref. ¹⁶ for a detailed discussion).

Finally, it is worth remarking on the measure of the summation over string world sheets in the above-mentioned theories. In the SVM, it is yet unknown as well as in the QCD itself and presumably defined by the perturbative fluctuations of the vacuum, mentioned in the Introduction. In 3D compact QED, as it has been argued in Ref. ⁹, the $\Sigma(C)$ -independence of $\langle W(C) \rangle$ is realized by the summation over branches of a certain multivalued potential of the monopole densities. The same is true ¹⁷ for the $SU(3)$ -inspired 3D Abelian-projected theory, where monopoles form a gas ¹⁸. In the 4D dual Abelian Higgs type theories, the summation over string world sheets appears from the integration over the multivalued part of the phase of the dual Higgs field, which is only nonvanishing on these world sheets. In particular, the Jacobian, which appears when one passes from the integration over this multivalued part to the integration over world sheets, has been evaluated in Ref. ¹⁹.

In the next Section, we shall concentrate ourselves on the applications of the above ideas to the evaluation of the bilocal field strength correlator, which plays the key rôle in the SVM. For concreteness, we shall evaluate it in the effective $SU(3)$ -inspired Abelian-projected theories in 4D and 3D.

3 The bilocal field strength correlator in the 4D $SU(3)$ -inspired dual Abelian Higgs type theory

In the London limit of infinitely heavy dual Higgs fields (*i.e.* infinitely thin dual strings), the Euclidean action of the model under study with an external

quark of the color c reads ²⁰

$$S_c = \int d^4x \left[\frac{1}{4} \left(\mathbf{F}_{\mu\nu} + \mathbf{F}_{\mu\nu}^{(c)} \right)^2 + \frac{\eta^2}{2} \sum_{a=1}^3 (\partial_\mu \theta_a - 2g_m \mathbf{e}_a \mathbf{B}_\mu)^2 \right]. \quad (3)$$

Here, g_m is the magnetic coupling constant, related to the QCD one as $g_m = \frac{4\pi}{g}$, $\mathbf{B}_\mu = (B_\mu^1, B_\mu^2)$ are the magnetic fields dual to the diagonal gluons $\mathbf{A}_\mu = (A_\mu^3, A_\mu^8)$, and η is the *v.e.v.* of the dual Higgs fields. The phases θ_a 's of the latter ones contain both the multivalued part, describing dual Nielsen-Olesen strings, and the single-valued one, describing perturbative fluctuations around them. Since monopole charges are distributed over the lattice defined by the root vectors \mathbf{e}_a 's of $SU(3)$, $\mathbf{e}_1 = (1, 0)$, $\mathbf{e}_2 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, $\mathbf{e}_3 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, the dual Higgs fields are not independent of each other, but their phases are rather subject to the constraint $\sum_{a=1}^3 \theta_a = 0$. The field of an external quark of the color $c = R, B, G$ (red, blue, green, respectively) is represented in Eq. (3) by the field strength tensor $\mathbf{F}_{\mu\nu}^{(c)}$, which obeys the equation $\partial_\mu \tilde{\mathbf{F}}_{\mu\nu}^{(c)} = \mathbf{Q}^{(c)} j_\nu^e$. Here, $j_\nu^e(x) \equiv g \oint_C dx_\nu(\tau) \delta(x - x(\tau))$, and $\mathbf{Q}^{(c)}$'s are the weights of the representation **3** of $^*SU(3)$, *i.e.* the charges of quarks *w.r.t.* the Cartan subgroup $[U(1)]^2$, which read $\mathbf{Q}^{(R)} = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$, $\mathbf{Q}^{(B)} = \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$, $\mathbf{Q}^{(G)} = \left(0, -\frac{1}{\sqrt{3}}\right)$.

The string representation of the action (3) has been derived in Refs. ^{21, 22} (see also Ref. ²³ for the generalization to the case when the Θ -term is included), and the resulting action reads

$$S_c = \pi^2 \int d^4x \int d^4y D_m^{(4)}(x - y) \left[\eta^2 \bar{\Sigma}_{\mu\nu}^a(x) \bar{\Sigma}_{\mu\nu}^a(y) + \frac{1}{6\pi^2} j_\mu^e(x) j_\mu^e(y) \right].$$

Here, $m = \sqrt{6}g_m\eta$ is the mass of the dual vector bosons, which they acquire due to the Higgs mechanism, and $D_m^{(4)}(x) \equiv \frac{m}{4\pi^2|x|} K_1(m|x|)$ is the respective propagator, where from now on K_ν 's stand for the modified Bessel functions. We have also introduced the following linear combinations of the vorticity tensor currents: $\bar{\Sigma}_{\mu\nu}^a \equiv \Sigma_{\mu\nu}^a - 2s_a^{(c)} \Sigma_{\mu\nu}^e$. Here, $\Sigma_{\mu\nu}^a(x) = \int_{\Sigma_a} d\sigma_{\mu\nu}(x_a(\xi)) \delta(x - x_a(\xi))$ is the vorticity tensor current defined on the closed string world sheet Σ_a , and $\Sigma_{\mu\nu}^e$ is the analogous expression defined on an arbitrary open string world sheet Σ^e bounded by the contour C . The numbers $s_a^{(c)}$'s read $s_3^{(R)} = s_2^{(B)} = s_1^{(G)} = 0$, $s_1^{(R)} = s_3^{(B)} = s_2^{(G)} = -s_2^{(R)} = -s_1^{(B)} = -s_3^{(G)} = 1$ and obey

the relation $\mathbf{Q}^{(c)} = \frac{1}{3}\mathbf{e}_a s_a^{(c)}$. Note also that owing to the constraint imposed on θ_a 's, $\Sigma_{\mu\nu}^a$'s also obey the constraint $\sum_{a=1}^3 \Sigma_{\mu\nu}^a = 0$, which should be imposed by the introduction of the respective δ -function into the partition function.

Following the SVM, let us further parametrize the bilocal correlator of electric field strengths $\mathbf{f}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$ as

$$\begin{aligned} \langle f_{\mu\nu}^i(x) f_{\lambda\rho}^j(0) \rangle_{\mathbf{A}_\mu, \mathbf{j}_\mu^m} &= \delta^{ij} \left\{ \left(\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda} \right) \hat{D}(x^2) + \right. \\ &\left. + \frac{1}{2} \left[\partial_\mu \left(x_\lambda \delta_{\nu\rho} - x_\rho \delta_{\nu\lambda} \right) + \partial_\nu \left(x_\rho \delta_{\mu\lambda} - x_\lambda \delta_{\mu\rho} \right) \right] \hat{D}_1(x^2) \right\}. \end{aligned} \quad (4)$$

Here, $\langle \dots \rangle_{\mathbf{A}_\mu}$ is the standard average over free gluons, and $\langle \dots \rangle_{\mathbf{j}_\mu^m}$ is a certain average over Abelian-projected monopoles, which are coupled to the field \mathbf{B}_μ dual to \mathbf{A}_μ . Referring the reader to Ref. ²⁴ for an exact form of the latter average, note here only that it describes the condensation of monopole Cooper pairs. By virtue of the parametrization (4), one has

$$\begin{aligned} -\ln \frac{\int \mathcal{D}x_\mu^a \delta \left(\sum_{a=1}^3 \Sigma_{\mu\nu}^a \right) e^{-S_c}}{\int \mathcal{D}x_\mu^a \delta \left(\sum_{a=1}^3 \Sigma_{\mu\nu}^a \right) e^{-S_c[C=0]}} &= \frac{1}{24} \int d^4x \int d^4y \times \\ &\times \left[2g^2 \Sigma_{\mu\nu}^e(x) \Sigma_{\mu\nu}^e(y) \hat{D}((x-y)^2) + j_\mu^e(x) j_\mu^e(y) \int_{(x-y)^2}^{+\infty} d\lambda \hat{D}_1(\lambda) \right]. \end{aligned} \quad (5)$$

Let us next specify the average over closed strings on the L.H.S. of Eq. (5). It is known (see *e.g.* ²⁵) that in the case of zero temperature under study such strings form virtual bound states consisting of a string and an antistring (*i.e.* two strings with opposite winding numbers), which are called vortex loops. The typical sizes of these objects are much smaller than the distances between them, which enables one to treat their grand canonical ensemble in the dilute gas approximation. The effective disorder field theory describing this ensemble has been constructed in Ref. ²⁶, and its action is given by the following formula:

$$S = \int d^4x \left\{ \frac{1}{12\eta^2} (H_{\mu\nu\lambda}^a)^2 + \frac{3}{2} g_m^2 (h_{\mu\nu}^a)^2 - \right. \\ \left. -2\zeta \left[\cos \left(\frac{\pi}{\Lambda^2 \sqrt{2}} \left| \sqrt{3} h_{\mu\nu}^1 + h_{\mu\nu}^2 \right| \right) + \cos \left(\frac{\pi}{\Lambda^2 \sqrt{2}} \left| \sqrt{3} h_{\mu\nu}^1 - h_{\mu\nu}^2 \right| \right) \right] \right\}. \quad (6)$$

Here, $H_{\mu\nu\lambda}^a \equiv \partial_\mu h_{\nu\lambda}^a + \partial_\lambda h_{\mu\nu}^a + \partial_\nu h_{\lambda\mu}^a$ is the field strength tensor of the Kalb-Ramond field ²⁷ $h_{\mu\nu}^a$. Next, $\zeta \propto e^{-S_0}$ is a Boltzmann factor of a single vortex loop with S_0 standing for its action, equal to the string tension of a loop times its area. We have also introduced an UV momentum cutoff $\Lambda \equiv \sqrt{\frac{L}{a^3}}$, where a denotes a typical size of the loop, whereas L stands for a typical distance between loops in the gas, so that $a \ll L$. The masses of the Kalb-Ramond fields following from Eq. (6) read $M_a^2 = m^2 + m_a^2$, where $m_1 = \frac{2\pi\eta}{\Lambda^2} \sqrt{3\zeta}$, $m_2 = \frac{2\pi\eta}{\Lambda^2} \sqrt{\zeta}$ are the contributions brought about by the Debye screening of the dual vector bosons in the gas of electric vortex loops.

By virtue of the representation of the effective theory (6) in terms of the integral over the densities of the vortex loops ^{26,24}, one can perform the average over the grand canonical ensemble of these objects on the L.H.S. of Eq. (5). Then, within the approximation that the typical sizes of vortex loops are completely negligible *w.r.t.* the area of Σ^e , one has ²¹ (see also Ref. ²⁸ for the respective $SU(2)$ -calculations):

$$\hat{D} = \frac{m^3}{4\pi^2} \frac{K_1(m|x|)}{|x|}, \quad (7)$$

$$\hat{D}_1 = \frac{m}{2\pi^2 x^2} \left[\frac{K_1(m|x|)}{|x|} + \frac{m}{2} \left(K_0(m|x|) + K_2(m|x|) \right) \right]. \quad (8)$$

In the IR limit, $|x| \gg \frac{1}{m}$, the asymptotic behaviors of the obtained coefficient functions read

$$\hat{D} \longrightarrow \frac{m^4}{4\sqrt{2}\pi^{\frac{3}{2}}} \frac{e^{-m|x|}}{(m|x|)^{\frac{3}{2}}}, \quad \hat{D}_1 \longrightarrow \frac{m^4}{2\sqrt{2}\pi^{\frac{3}{2}}} \frac{e^{-m|x|}}{(m|x|)^{\frac{5}{2}}}. \quad (9)$$

Comparing now Eqs. (9) with the results of the lattice investigations ^{4,5,6,7}, one can easily see that they match each other. In particular, it is clearly seen that for the bilocal correlator of gluonic field strengths of the same kind, the vacuum of the model (3) exhibits a nontrivial correlation length $T_g = \frac{1}{m}$.

Moreover, it is possible to improve on the results (7) and (8) by keeping the size of the vortex loops finite. Then, taking into account the contributions of the bilocal correlators of the vortex loops to the coefficient functions \hat{D} and \hat{D}_1 , one arrives at the following modifications of these functions ²⁴:

$$\hat{D} = \frac{m^2 M_2}{4\pi^2} \frac{K_1(M_2|x|)}{|x|}, \quad (10)$$

$$\hat{D}_1 = \frac{m_2^2}{\pi^2 M_2^2 |x|^4} + \frac{m^2}{2\pi^2 M_2 x^2} \left[\frac{K_1(M_2|x|)}{|x|} + \frac{M_2}{2} (K_0(M_2|x|) + K_2(M_2|x|)) \right]. \quad (11)$$

It is straightforward to see that when m_2 vanishes, *i.e.* one disregards the effect of screening of the dual vector bosons by the vortex loops, the old expressions (7) and (8) for the functions \hat{D} and \hat{D}_1 are recovered. We also see that since the screening enhances the mass of the dual vector bosons, the correlation length of the vacuum becomes also modified from $\frac{1}{m}$ to $\frac{1}{M_2}$. The asymptotic behaviors (9) change respectively to

$$\hat{D} \longrightarrow \frac{(mM_2)^2}{4\sqrt{2}\pi^{\frac{3}{2}}} \frac{e^{-M_2|x|}}{(M_2|x|)^{\frac{3}{2}}}, \quad \hat{D}_1 \longrightarrow \frac{m_2^2}{\pi^2 M_2^2 |x|^4} + \frac{(mM_2)^2}{2\sqrt{2}\pi^{\frac{3}{2}}} \frac{e^{-M_2|x|}}{(M_2|x|)^{\frac{3}{2}}}.$$

It is remarkable that the screening leads to the appearance of the IR power-like part of the function \hat{D}_1 . On the other hand, one can check that the UV asymptotic behaviors of the functions \hat{D} and \hat{D}_1 remain unaffected by the screening, as well as the string tension.

4 The bilocal field strength correlator in the 3D-gas of $SU(3)$ Abelian-projected monopoles

The partition function of the 3D grand canonical ensemble of $SU(3)$ Abelian-projected monopoles reads ¹⁸

$$\mathcal{Z} = 1 + \sum_{N=1}^{\infty} \frac{\zeta^N}{N!} \left(\prod_{n=1}^N \int d^3 z_n \sum_{a_n=\pm 1, \pm 2, \pm 3} \right) \exp \left[-\frac{g_m^2}{4\pi} \sum_{n < k} \frac{\mathbf{e}_{a_n} \mathbf{e}_{a_k}}{|\mathbf{z}_n - \mathbf{z}_k|} \right]. \quad (12)$$

Here, $\zeta \propto \exp\left(-\frac{\text{const}}{g^2}\right)$ is now the Boltzmann factor of a single monopole, and $\mathbf{e}_{-a} = -\mathbf{e}_a$. The action of the respective disorder field theory has the form ¹⁷

$$S = \int d^3x \left\{ \frac{1}{2}(\nabla\chi_1)^2 + \frac{1}{2}(\nabla\chi_2)^2 - \right. \\ \left. - 2\zeta \left[\cos(g_m\chi_1) + \cos\left(\frac{g_m}{2}(\chi_1 + \sqrt{3}\chi_2)\right) + \cos\left(\frac{g_m}{2}(\chi_1 - \sqrt{3}\chi_2)\right) \right] \right\}. \quad (13)$$

According to this expression, the Debye masses of the dual fields χ_1 and χ_2 are equal to each other and read $m = g_m\sqrt{3}\zeta$.

To discuss the field strength correlators in the model (13), it is useful to derive the string representation for the monopole part of the Wilson loop. The result has the form of an integral over the monopole densities \mathbf{j} 's and reads¹⁷

$$\langle W(C) \rangle_m = \frac{1}{3\mathcal{Z}} \sum_{c=R,B,G} \int \mathcal{D}\mathbf{j} \times \\ \times \exp \left\{ - \left[\frac{2\pi}{g^2} \int d^3x \int d^3y \mathbf{j}(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{y}|} \mathbf{j}(\mathbf{y}) + V[\mathbf{j}] - i \int d^3x \mathbf{j} \mathbf{Q}^{(c)} \eta \right] \right\}. \quad (14)$$

Here,

$$V[\mathbf{j}] = \sum_{n=-\infty}^{+\infty} \sum_{\alpha=1}^3 \int d^3x \times \\ \times \left\{ j_\alpha \left[\ln \left(\frac{j_\alpha}{2\zeta} + \sqrt{1 + \left(\frac{j_\alpha}{2\zeta} \right)^2} \right) + 2\pi i n \right] - 2\zeta \sqrt{1 + \left(\frac{j_\alpha}{2\zeta} \right)^2} \right\} \quad (15)$$

is the effective multivalued potential of the monopole densities with $j_1 \equiv \frac{1}{\sqrt{3}} \left(\frac{1}{\sqrt{3}} j^1 + j^2 \right)$, $j_2 \equiv -\frac{2}{3} j^1$, $j_3 \equiv \frac{1}{\sqrt{3}} \left(\frac{1}{\sqrt{3}} j^1 - j^2 \right)$. In Eq. (14), we have also denoted by η the solid angle, under which the surface $\Sigma(C)$ shows up to an observer located at the point \mathbf{x} ,

$$\eta[\mathbf{x}, \Sigma(C)] \equiv \frac{1}{2} \varepsilon_{\mu\nu\lambda} \frac{\partial}{\partial x_\mu} \int_{\Sigma(C)} d\sigma_{\nu\lambda}(\mathbf{y}) \frac{1}{|\mathbf{x} - \mathbf{y}|}.$$

Notice that similarly to the case of compact QED⁹, it is the summation over the branches of the potential (15), which restores the $\Sigma(C)$ -independence of

$\langle W(C) \rangle_{\text{m}}$. Furthermore, one can restrict oneself to the real branch of this potential by performing the replacement $\Sigma(C) \rightarrow \Sigma_{\text{min}}(C) \equiv \Sigma_{\text{min}}$. Then, in the dilute monopole gas approximation, $|\mathbf{j}| \ll \zeta$, it is straightforward to carry out the resulting Gaussian integration over the monopole densities. Combining the so-obtained result with the contribution to the Wilson loop stemming from the free diagonal gluons,

$$\langle W(C) \rangle_{\text{free}} = \exp \left(-\frac{g^2}{24\pi} \oint_C dx_\mu \oint_C dy_\mu \frac{1}{|\mathbf{x} - \mathbf{y}|} \right),$$

we obtain the following result for the full Wilson loop:

$$\begin{aligned} \langle W(C) \rangle &= \langle W(C) \rangle_{\text{m}} \langle W(C) \rangle_{\text{free}} = \\ &= \exp \left\{ - \left[\pi\zeta \int_{\Sigma_{\text{min}}} d\sigma_{\mu\nu}(\mathbf{x}) \int_{\Sigma_{\text{min}}} d\sigma_{\mu\nu}(\mathbf{y}) + \frac{g^2}{24\pi} \oint_C dx_\mu \oint_C dy_\mu \right] \frac{e^{-m|\mathbf{x}-\mathbf{y}|}}{|\mathbf{x} - \mathbf{y}|} \right\}. \end{aligned}$$

Parametrizing further the bilocal correlator of the field strengths according to Eq. (4) (where the average $\langle \dots \rangle_{\text{j}_\mu}$ is now replaced by the average over the dilute 3D gas of monopoles) with the redefinitions $\hat{D} \rightarrow \mathcal{D}$, $\hat{D}_1 \rightarrow \mathcal{D}_1$, we obtain:

$$\mathcal{D} = 12\pi\zeta \frac{e^{-m|\mathbf{x}|}}{|\mathbf{x}|}, \quad \mathcal{D}_1 = \frac{24\pi\zeta}{(m|\mathbf{x}|)^2} \left(m + \frac{1}{|\mathbf{x}|} \right) e^{-m|\mathbf{x}|}.$$

Similarly to the 4D-case considered in the previous Section, one can see a good correspondence of these results with those obtained on the lattice for the real QCD in Refs. ^{4, 5, 6, 7}. In particular, the rôle of the correlation length of the vacuum is played in the model under study by the inverse Debye mass, $\frac{1}{m}$, and at the distances $|\mathbf{x}| \gg \frac{1}{m}$, $\mathcal{D} \gg \mathcal{D}_1$ due to the preexponential behavior.

5 Conclusions

In this talk, we have addressed the topic of a derivation of the bilocal field strength correlators in various $SU(3)$ -inspired Abelian-projected theories. Those included the 4D dual Abelian Higgs type theory and the 3D theory, in which Abelian-projected monopoles were assumed to form a gas. In both

cases, the derivation of the field strength correlators essentially employed the ideas of the string representation of the respective theories. In particular, in the first case the contribution of the bilocal correlators of vortex loops, formed by the closed dual strings with opposite winding numbers, to the correlator of the field strengths was accounted for. This led to the modification of the correlation length of the vacuum in the model under study. Namely, it changed from the classical expression, equal to the inverse mass of the dual vector bosons generated by the Higgs mechanism, to the inverse enhanced mass. The latter one took into account the effect of the Debye screening of these bosons in the gas of electric vortex loops. Besides that, it turned out that the screening yielded also the IR power-like contribution to one of the two coefficient functions, which parametrized the bilocal correlator of the field strengths.

In the second case, the derivation of the field strength correlator was essentially based on the string representation for the Wilson loop in the 3D Coulomb gas of the $SU(3)$ Abelian-projected monopoles. Similarly to the 4D dual Abelian Higgs type theory, in this case the results for the coefficient functions, which parametrize the bilocal correlator of the field strengths, match the results of lattice measurements in QCD with a good accuracy. In both cases, only *w.r.t.* the correlators of the field strengths built out of the gluon fields corresponding to the same generator of the Cartan algebra the vacua of the considered theories exhibited a nontrivial correlation length.

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